Digital to Analog Converters (DAC)

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Outline

- Purpose
- Types
- Performance Characteristics
- Applications
Purpose

- To convert digital values to analog voltages
- Performs inverse operation of the Analog-to-Digital Converter (ADC)

\[ V_{OUT} \propto \text{Digital Value} \]
DACs

**Types**
- Binary Weighted Resistor
- R-2R Ladder
- Multiplier DAC
  - The reference voltage is constant and is set by the manufacturer.
- Non-Multiplier DAC
  - The reference voltage can be changed during operation.

**Characteristics**
- Comprised of switches, op-amps, and resistors
- Provides resistance inversely proportion to significance of bit
Binary Weighted Resistor

\[ R_f = R \]

\[ \sum I_i \]

- \( V_{\text{REF}} \)

MSB

LSB
Binary Representation

Most Significant Bit

Least Significant Bit

R
2R
4R
8R

\[ R_f = R \]

\[ \sum I_i \]

\[ V_o \]

\[ -V_{REF} \]
Binary Representation

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
\end{array}
\]

\[= 15_{10}\]
Binary Weighted Resistor

- “Weighted Resistors” based on bit
- Reduces current by a factor of 2 for each bit
Binary Weighted Resistor

Result:

\[ \sum I = V_{REF} \left( \frac{B_3}{R} + \frac{B_2}{2R} + \frac{B_1}{4R} + \frac{B_0}{8R} \right) \]

\[ V_{OUT} = I \cdot R_f = V_{REF} \left( B_3 + \frac{B_2}{2} + \frac{B_1}{4} + \frac{B_0}{8} \right) \]

- \( B_i = \) Value of Bit \( i \)
Binary Weighted Resistor

More Generally:

\[ V_{OUT} = V_{REF} \sum \frac{B_i}{2^{n-i-1}} \]

\[ = V_{REF} \cdot \text{Digital Value} \cdot \text{Resolution} \]

- \( B_i \) = Value of Bit i
- \( n \) = Number of Bits
R-2R Ladder
R-2R Ladder

- Same input switch setup as Binary Weighted Resistor DAC
- All bits pass through resistance of 2R
R-2R Ladder

- The less significant the bit, the more resistors the signal must pass through before reaching the op-amp.
- The current is divided by a factor of 2 at each node.

![Diagram of an R-2R ladder network](image)
R-2R Ladder

- The current is divided by a factor of 2 at each node
- Analysis for current from (001)_2 shown below

\[ I_0 = \frac{-V_{REF}}{2R + 2R|2R} = \frac{V_{REF}}{3R} \]
R-2R Ladder

- Result:

\[ I = \frac{V_{REF}}{3R} \left( \frac{B_2}{2} + \frac{B_1}{4} + \frac{B_0}{8} \right) \]

\[ V_{OUT} = \frac{R_f}{R} V_{REF} \left( \frac{B_2}{2} + \frac{B_1}{4} + \frac{B_0}{8} \right) \]

- \( B_i = \) Value of Bit i
R-2R Ladder

- If $R_f = 6R$, $V_{OUT}$ is same as Binary Weighted:

$$I = \frac{V_{REF}}{3R} \sum \frac{B_i}{2^{n-i}}$$

$$V_{OUT} = V_{REF} \sum \frac{B_i}{2^{n-i-1}}$$

- $B_i = \text{Value of Bit } i$
R-2R Ladder

Example:
- Input = (101)_2
- \( V_{REF} = 10 \text{ V} \)
- \( R = 2 \, \Omega \)
- \( R_f = 2R \)

\[
I_0 = \frac{-V_{REF}}{2R + 2R || 2R} = \frac{V_{REF}}{3R} = -1.67 \text{ mA}
\]

\[
I_{\text{op-amp}} = \frac{I_0}{8} + \frac{I_0}{2} = -1.04 \text{ mA}
\]

\[
V_{OUT} = -I_{\text{op-amp}} R_f = 4.17 \text{ V}
\]
# Pros & Cons

<table>
<thead>
<tr>
<th>Pros</th>
<th>Binary Weighted</th>
<th>R-2R</th>
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<tr>
<td><strong>Pros</strong></td>
<td>Easily understood</td>
<td>Only 2 resistor values</td>
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<tr>
<td><strong>Cons</strong></td>
<td>Limited to ~ 8 bits</td>
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<td>Large # of resistors</td>
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<td>Susceptible to noise</td>
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<td></td>
<td>Expensive</td>
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<td>Greater Error</td>
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Digital to Analog Converters

- Performance Specifications
- Common Applications

Presented by: Mark Hunkele
Digital to Analog Converters

- **Performance Specifications**

- Resolution
- Reference Voltages
- Settling Time
- Linearity
- Speed
- Errors
Resolution: is the amount of variance in output voltage for every change of the LSB in the digital input.

How closely can we approximate the desired output signal (Higher Res. = finer detail = smaller Voltage divisions)

A common DAC has a 8 - 12 bit Resolution

\[
\text{Resolution} = V_{LSB} = \frac{V_{\text{Ref}}}{2^N} \quad \text{N = Number of bits}
\]
Digital to Analog Converters

- Performance Specifications
  - Resolution

**Poor Resolution (1 bit)**

- **Desired Analog signal**
- **Approximate output**

**Better Resolution (3 bit)**

- **Desired Analog signal**
- **Approximate output**
Reference Voltage: A specified voltage used to determine how each digital input will be assigned to each voltage division.

Types:
- Non-multiplier: internal, fixed, and defined by manufacturer
- Multiplier: external, variable, user specified
Assume 2 bit DAC

Non-Multiplier: \( V_{\text{ref}} = C \)

\[
\begin{array}{c|c|c|c|c}
\text{Digital Input} & 00 & 01 & 10 & 11 \\
\hline
\text{Voltage} & 0 & C/4 & C/2 & 3C/4 \\
\end{array}
\]

Multiplier: \( V_{\text{ref}} = A\sin(wt) \)

\[
\begin{array}{c|c|c|c|c}
\text{Digital Input} & 00 & 01 & 10 & 11 \\
\hline
\text{Voltage} & 0 & A/4 & A/2 & 3A/4 \\
\end{array}
\]
Settling Time: The time required for the input signal voltage to settle to the expected output voltage (within $\pm V_{\text{LSB}}$).

Any change in the input state will not be reflected in the output state immediately. There is a time lag, between the two events.
Digital to Analog Converters

-Performance Specifications

-Settling Time

Analog Output Voltage

Expected Voltage

Settling time

Time

+V_{\text{LSB}}

-V_{\text{LSB}}
Digital to Analog Converters

-Performance Specifications

-Linearity

- Linearity: is the difference between the desired analog output and the actual output over the full range of expected values.

- Ideally, a DAC should produce a linear relationship between a digital input and the analog output, this is not always the case.
Digital to Analog Converters
-Performance Specifications
-Linearity

Linearity (Ideal Case)

- Perfect Agreement

NON-Linearity (Real World)

- Miss-alignment
Digital to Analog Converters

- Performance Specifications

-Speed

- **Speed**: Rate of conversion of a single digital input to its analog equivalent

- **Conversion Rate**
  - Depends on clock speed of input signal
  - Depends on settling time of converter
Digital to Analog Converters

- Performance Specifications

- Errors

- Non-linearity
  - Differential
  - Integral

- Gain

- Offset

- Non-monotonicity
Digital to Analog Converters

- Performance Specifications

**-Errors: Differential Non-Linearity**

- **Differential Non-Linearity**: Difference in voltage step size from the previous DAC output (Ideally All DLN’s = 1 $V_{LSB}$)

![Diagram showing differential non-linearity concept with steps and linearity comparison.](image-url)
Digital to Analog Converters
-Performance Specifications

**-Errors: Integral Non-Linearity**

- **Integral Non-Linearity**: Deviation of the actual DAC output from the ideal (Ideally all INL’s = 0)

![Diagram showing integral non-linearity](image)
Digital to Analog Converters

- Performance Specifications

**Errors: Gain**

- **Gain Error**: Difference in slope of the ideal curve and the actual DAC output

  - **High Gain Error**: Actual slope greater than ideal
  - **Low Gain Error**: Actual slope less than ideal
Offset Error: A constant voltage difference between the ideal DAC output and the actual.

- The voltage axis intercept of the DAC output curve is different than the ideal.
Digital to Analog Converters

-Performance Specifications

**Errors: Non-Monotonicity**

- **Non-Monotonic**: A decrease in output voltage with an increase in the digital input.

![Diagram showing monotonic and non-monotonic behavior](image)
Digital to Analog Converters

- Common Applications

- Generic use
- Circuit Components
- Digital Audio
- Function Generators/Oscilloscopes
- Motor Controllers
Digital to Analog Converters

- Common Applications

- **Generic**

- Used when a continuous analog signal is required.

- Signal from DAC can be smoothed by a Low pass filter

![Diagram of digital to analog conversion process]

Digital Input → n bit DAC → Piece-wise Continuous Output → Filter → Analog Continuous Output
Digital to Analog Converters

- Common Applications

-Circuit Components

- Voltage controlled Amplifier
  - digital input, External Reference Voltage as control

- Digitally operated attenuator
  - External Reference Voltage as input, digital control

- Programmable Filters
  - Digitally controlled cutoff frequencies
Digital to Analog Converters

- Common Applications

- Digital Audio

- CD Players
- MP3 Players
- Digital Telephone/Answering Machines

Digital to Analog Converters

- Common Applications
  - Function Generators

- Digital Oscilloscopes
  - Digital Input
  - Analog Output

- Signal Generators
  - Sine wave generation
  - Square wave generation
  - Triangle wave generation
  - Random noise generation

Digital to Analog Converters

- Common Applications
  - Motor Controllers

- Cruise Control
- Valve Control
- Motor Control

References

- “Simplified DAC/ADC Lecture Notes,” http://www-personal.engin.umich.edu/~fmeral/ELECTRONICS II/ElectronicII.html